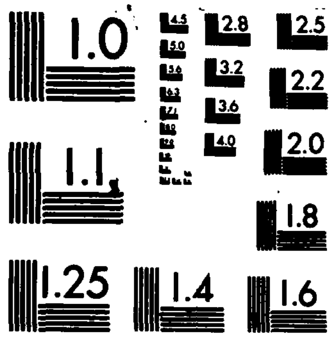

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THE ANALYSIS OF ELASTIC-PLASTIC DEFORMATION AND STRESS AT FINITE STRAIN AND THEIR EVALUATION

Metal Forming Report No. 8/83

FINAL REPORT

E. H. Lee and R. L. Mallett

December 30, 1983

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ABSTRACT

The facility of structural metals to be deformable to large strains without fracturing is often needed in the manufacturing of structural components (metal forming) and in the operation of a structure (e.g. armor penetration, crashworthiness and earthquake resistance). Rational design of a structure therefore often demands stress and deformation analysis in the presence of finite strain.

Investigations of several aspects of elastic-plastic theory at finite deformation have been carried out on this project including the nonlinear coupling of elastic and plastic strains when the deformation is large, the representation and analysis of plastic strain-induced anisotropy and the evaluation of stress distributions in forming processes by the finite-element method.

Of particular interest and importance, because what was until recently thought to be the correct formulation of the theory turned out to introduce huge errors in stress analysis, is the prediction of stress and deformation distributions in material exhibiting plastic strain-induced anisotropy of the type associated with the Bauschinger effect. State-of-the-art calculations presented at a workshop, co-sponsored by ARO, predicted oscillating shear stress generated by monotonically increasing shear strain to large strains. This anomaly turned out to be caused by an incorrect formulation of the influence of material rotation on the growth of the induced anisotropy. The latter is caused by a distribution of residual stress generated by the polycrystalline structure of the material considered. The residual stress field is embedded in the material at the crystallite level and rotates with it at a rate determined by the combination of spin and rate of deformation occurring. A promising first approach towards correcting the error has been devised on the basis of a simple hypothesis for the macroscopic influence of the micro-mechanisms that generate the hardening. A more thorough study of this aspect of the theory is called for. The shortcomings of an approach suggested by other investigators of using the polar-decomposition rotation to prescribe the material rotation influence on the anisotropy is pointed out.

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THE ANALYSIS OF ELASTIC-PLASTIC DEFORMATION AND STRESS AT FINITE STRAIN AND THEIR EVALUATION

INTRODUCTION

The facility of structural metals to be deformable to large strains without fracturing is often needed in the manufacturing of structural components (metal forming) and in the operation of a structure (e.g. armor penetration, crash-worthiness and earthquake resistance). Rational design of a structure therefore often demands stress and deformation analysis in the presence of finite strain. Furthermore, in order to anticipate and hence avoid the generation of cracks, strain localization, high residual stresses or other forming defects, it may be necessary to evaluate stress throughout a body because the location of the material element in which a defect might be initiated is not known in advance and the total history of stress there may be needed to predict the phenomenon. Elastic-plastic theory must thus be utilized, for neglect of the small elastic strain compared with plastic strain, and hence the adoption of rigid-plastic theory, would prevent the determination of stresses in the rigid regions which may well comprise most of the work-piece in a metal forming process.

This project was concerned with the formulation of stress-deformation relations at finite strain and rotation in particular for materials in which plastic flow generated anisotropic plastic characteristics of the type associated with the Bauschinger effect. It also addressed the general structure of finite-deformation elastic-plastic constitutive relations based on strict uncoupling of the elastic and plastic responses according to the physical phenomena which generate them. A computer program was further

developed to evaluate stress and deformation distributions in metal forming processes. References [1] - [6] comprise papers and manuscripts issued on this project. References [7] and [8] are papers generated on previous ARO grants published during the period of this grant and the volume [9] is the proceedings of the Workshop on Finite Deformation Plasticity co-sponsored by ARO, published in 1982.

Some highlights of research findings achieved on this project are summarized below.

STRESS ANALYSIS FOR PLASTIC-STRAIN-INDUCED ANISOTROPY

An unexpected outcome of the Workshop: Plasticity of Metals at Finite Strain [9], co-sponsored by ARO, was the realization that the then state-of-the-art computer programs for stress analysis at finite strain of materials which exhibit plastic strain-induced anisotropy (e.g. the Bauschinger effect) predicted spurious oscillation of the shear stress generated by monotonically increasing shear strain. This involved huge errors in the calculated stress. The difficulty was not resolved during the workshop, but it later became clear [3] that erroneous mathematical representation of the influence of material rotation on the growth of the anisotropy was responsible for the anomaly.

The plastic anisotropy was caused by a residual stress distribution, called the back stress $\underset{\sim}{\alpha}$ generated by deformation of the poly-crystalline material comprising a more or less random agglomeration of crystallites, each exhibiting the crystallographic anisotropy of a single crystal. Such residual stresses, which strengthen a structure for continued loading in the direction previously applied but reduce the yielding strength in reverse loading, commonly arise due to the development of non-homogeneous plastic strain, as

for example in autofrettage of a gun tube or pressure vessel. In the case of plastic yielding of a poly-crystalline material, the effect of this residual stress distribution is expressed as a kinematic hardening law with a shift $\underset{\sim}{\alpha}$ of the yield surface in stress space. The residual stresses vary appreciably from crystallite to crystallite in such a way that the macroscopic stress is zero. Those crystallites oriented so that they are amongst the first to flow plastically during the initial loading will have accumulated residual stresses of the opposite sign and thus on repeated loading in the original direction renewed plastic flow will be delayed. In contrast, reverse loading will experience a reduced yield stress. Averaged over a surface which intersects many crystallites, the magnitude of the residual stresses will approach zero since this average must approach the macroscopic stress. Thus the physical mechanism causing the anisotropy of the yield stress is embedded in the material on the crystallite scale.

In simple shearing deformation, material elements distort and rotate causing growth and rotation of the residual stresses and hence of the macroscopic plastic anisotropy which they cause. Because distortion and rotation are occurring simultaneously, different material lines through a material point rotate with different angular velocities, the average value of which is the spin $\underset{\sim}{W}$.

A study of the micromechanics of the situation, either at the crystallite level, the dislocation level, or at both, may be needed to fully understand this question, but this has not yet been done. However, information can be gleaned from the macroscopic theory. A physically based model which provides a first approximation to the effect of material rotation on the evolution of the anisotropy and includes major mechanisms which govern the phenomenon was

presented in [3]. It yielded promising results and eliminated the anomaly generated by the then state-of-the-art codes. The stress field exhibited a reduction of the rate of hardening as the direction of maximum anisotropy rotated away from the direction of maximum strain rate, a phenomenon known as the rotational Bauschinger effect.

Alerted to the problem by an earlier version of [3] issued in report form, a different approach to the inclusion of the material rotation influence on the evolution of the plastic strain-induced anisotropy has been suggested by some investigators. It hypothesizes rotation of the anisotropy by the polar-decomposition rotation of the total deformation, and is being applied by groups associated with defense activity. However, as explained below, this approach introduces severe shortcomings in failing to model the physical mechanism which causes the plastic anisotropy. As has been described, the residual stress distribution which embodies the anisotropy is embedded in the poly-crystalline material and this elastic interaction between material elements lined-up in certain directions and generated by previous plastic flow plays a predominant role in determining the characteristics of the plastic anisotropy. It is the joint influence of the rotation of these directions which determines the resultant rotation of the macroscopic anisotropic properties. Since the micromechanics at the crystallite level has not been fully analysed, it is instructive to consider an analogous structure, filamentary composite material, which provides a simpler model illustrating the formulation of the influence of material rotation (the orientations of the embedded filaments constitute a more easily tracked entity causing the anisotropy).

Consider, for example, the composite shown in Fig. 1, consisting of an ideally plastic material with inextensible filaments lying parallel to the X_1 axis. Simple shear γ takes place defined by

$$x_1 = X_1 + \gamma X_2$$
$$x_2 = X_2$$

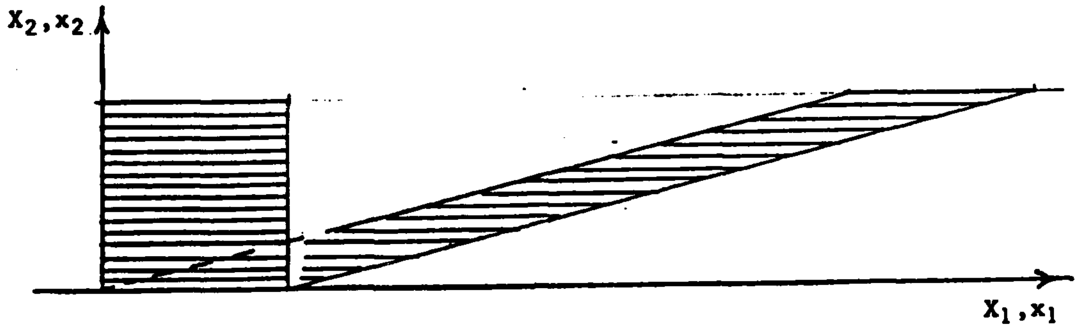

Fig. 1. - Simple shear of a filamentary composite material.

where X_i are the reference coordinates prescribing the initial condition of the body and x_i expresses the deformed position. It is clear from Fig. 1 that under this deformation the anisotropy does not change its orientation. However, the polar-decomposition rotation is of magnitude Θ, where $\tan \Theta = \gamma/2$*, and hence Θ approaches $\pi/2$ for large γ. Figure 2 shows the form of the deformation expressed by polar decomposition into a sequence of pure rotation and pure

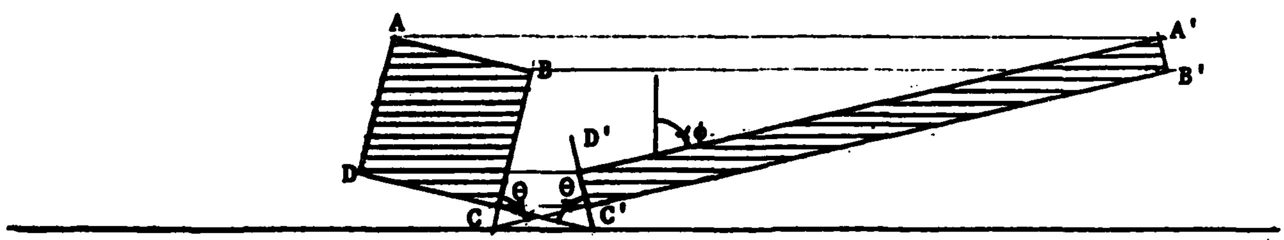

Fig. 2. - Simple shear of the composite decomposed into pure deformation and rotation.

*See the Appendix

stretch along principal axes. Thus the hypothesis that the polar-decomposition rotation constitutes the "material" rotation which is therefore the rotation of the anisotropy clearly fails to express what is happening physically.

The theory developed in [3] attempts to formulate a physical model of the cause of the anisotropy which in the case of the simple composite illustrated in Fig. 1 would delineate the direction of the filaments as that carrying the anisotropy. Since the filaments do not rotate under the deformation considered, the model tells us that the orientation of the anisotropy does not change. The preliminary theory in [3] was suggested in the absence of a rigorous micro-mechanical analysis; in contrast the polar-decomposition model was offered as an exact approach. However, the composite example shows that such a purely mathematical model can introduce serious error because it fails to include any variables expressing the characteristics of the existing anisotropy, and these can have a major bearing on the influence of material rotation on the evolution of the anisotropy. Plastic anisotropy implies that certain directions at a material point have special significance with regard to material response to deformation and during deformation the angular velocity of lines of material elements depends on their orientation, thus no averaged angular velocity, whether spin (the anti-symmetric part of the velocity gradient) or the polar-decomposition spin can in general express the rotation effect of deformation on the evolution equation for the back stress in the anisotropic hardening problem. Reference [3] selects a significant direction associated with the anisotropy and the angular velocity of the line of material elements determined by that direction which carries a major component of the anisotropy

contributes the material rotation influence on the evolution equation for the back stress.

For an isotropic non-linear elastic material, the orientation of the induced anisotropy due to deformation (in the sense of anisotropic response for superposed small additional straining) will have principal directions coincident with those of the stress and the strain i.e. parallel to A'B' and B'C' in Fig. 2 (in this case, of course, the filaments shown should be ignored since they are relevant only for the filamentary composite case). The principal directions are not in a strict sense tied to or embedded in the material for they are determined directly from the current stress by a function law, specifically the isotropic elastic law. The inclination of the direction C'B' or D'A' from the X_2 direction, ϕ, is given by $\tan\phi = \sqrt{1+\gamma^2/4} + \gamma/2$. Initially the polar decomposition rotation $\Theta = 0$ whereas $\phi = \pi/4$ but for large γ both approach $\pi/2$. They approach $\pi/2$ at different rates since $\tan(\pi/2 - \Theta) = 2/\gamma$ and $\tan(\pi/2 - \phi) \to 1/\gamma - 1/(\gamma)^3$. The direction of anisotropy thus rotates with angular velocity $\dot{\phi}$ which is not equal to the polar-decomposition spin $\dot{\Theta}$, equivalent to $\dot{\underset{\sim}{R}}\,\underset{\sim}{R}^T$, but is directly related to it through γ. Deformation type plasticity theory would respond in the same way, but is well known to be unsatisfactory for plasticity in other than proportional or approximately proportional loading. This approach is therefore unsuitable for a general purpose computer program which must be valid for arbitrary stress imputs for the finite elements.

In view of the developments already described of attempts to incoporate strain induced anisotropy into plastic stress analysis at finite deformation, it is clearly imperative to correct and/or refine approaches that have been

initiated. Moreover, it will be necessary to extend the investigations of strain induced anisotropy to that associated with change of shape of the yield surface in addition to the shift (kinematic hardening) discussed in this report. Analogous rotation influences can be expected in the former. Valid physical models of the phenomena are needed and experimental data for general loading histories. Studies already carried out on the Bauschinger effect, as well as studies concerning plasticity of single crystals, the inte. c'on in poly-crystalline aggregates and macroscopic elastic-plastic contin theory at finite deformation, may provide information not already in -ated into the study of this problem. Developments in all these areas will be needed to resolve this applied-mechanics problem which is important in many engineering applications, particularly in the area of armaments design.

ELASTIC-PLASTIC THEORY AT FINITE STRAIN

Rate-independent incremental elastic-plastic theory is an ideal vehicle for learning to construct an analytical framework involving the combination of two different types of material behavior, for elastic and plastic characteristics are so disparate that an inappropriate combined structure is likely to provide clear evidences of mechanical inadequacy. Elasticity generates a function law between stress and strain which is reversible. Plasticity is governed by an irreversible dissipative functional law in which the stress depends on the history of the strain and vice-versa. Physically the phenomena are quite different for metals (our main concern), elasticity comprises distortion of the crystal lattice whereas plasticity is associated with slip over crystallographic slip planes. Thus elastic strain is directly related to the stress and plastic strain is the strain remaining when the stress is reduced to zero so that the elastic-strain component is zero. If plastic flow occurs

on unloading, measurement of the elastic-strain-energy function in the elastic region permits a plastic strain to be deduced such that the structure of the theory is identical with that corresponding to elastic unloading and gives exact results in the elastic region, i.e. wherever it is needed in a physical problem.

Although the above description of elastic-plastic theory suggests a simple means of uncoupling the elastic and plastic components of deformation, the fact that the plasticity law is incremental in nature (equivalently termed "of flow type") demands use of strain increments or strain rates as variables. For finite strains these involve a nonlinear coupling between the elastic and plastic contributions, also involving the elastic strain. It has been shown [10] that, in the case of isotropic elastic response and isotropic hardening, a derivative of the elastic constitutive relation valid for finite strain can be combined with the plasticity law for rate of plastic strain and the nonlinear kinematic relation for total strain rate in terms of elastic and plastic components to yield an elastic-plastic constitutive relation for the total strain rate in terms of the stress rate and deformation history. This procedure is a purely deductive mathematical process. Thus the physical characteristics of elasticity and plasticity are embodied in terms of the elastic and plastic deformations, and the incremental nature of the elastic-plastic constitutive relation is incorporated through use of the incremental plasticity law and the formally differentiated elasticity law.

In contrast, the usual approach is to assume that the strain rate is summable in terms of elastic and plastic strain rates. This ignores some elastic-plastic coupling as becomes evident when the kinematics of strain rates is deduced from the physically based definitions of elastic and plastic

deformations presented above [11]. Moreover, the usual use of Hooke's law for the elastic relation, because elastic strains are commonly considered to remain small, cannot be formally differentiated to produce a rate form since it is not objective with respect to the finite rotations commonly arising. The usual approach is to form an elastic rate law by expressing the strain rate in terms of the velocity field associated with elastic deformation and to replace the stress rate by an objective derivative of stress to be selected from the infinite number available. The logical structure of the procedure fails because of the lack of precision in expressing elastic and plastic components and the limitations of the mathematical structure of Hooke's law. These shortcomings limit the precision with which work theorems and variational principles can be formulated, restrictions not shared by the nonlinear kinematic theory.

Since the small-elastic-deformation theory based on the summability of elastic and plastic strain rates poses a simpler computational task and in most problems likely to arise in engineering practice is a close approximation to the more complex theory based on nonlinear kinematics, the former is usually the appropriate choice for stress evaluations. However, some problems, for example high pressure impact and explosively generated waves involve large elastic dilatational strains which require the finite-elastic-strain analysis. The more complete theory may also be needed for the study of instabilities and localization of deformation since such problems are particularly sensitive to minor variations in the constitutive relations.

DEVELOPMENT OF THE FINITE-ELEMENT METHOD FOR ELASTIC-PLASTIC STRESS ANALYSIS

The finite-element computer program, IFDEPSA (an acronym for Incremental Finite Deformation Elastic-Plastic Stress Analyzer), was further developed on

this project to carry out the computations required for elastic-plastic deformation and stress analysis at finite strain. IFDEPSA was first used to compute the residual stresses generated in plane strain extrusion of a metal through frictionless curved dies. In order to extend the program's capability of analyzing practical metal forming processes, it was necessary to develop improved computational algorithms and procedures and to develop the capabilities of modeling complex metal-tool interface boundary conditions and more complex constitutive behavior. Considerable progress has been made in all three areas. Numerical procedures were developed, for example, to avoid biased errors usually associated with incremental analysis, such as drift from the yield surface. In [7] it was demonstrated that two relatively simple types of four noded finite elements are much more efficient and accurate than higher order elements in dealing with the basic problem of the incompressibility of plastic flow. Boundary friction was successfully treated and some progress in handling the complex boundary conditions associated with the rolling process has been achieved. The capability of treating anisotropic plastic material behavior has also been implemented.

CONCLUSION AND RECOMMENDATION

From the standpoint of correcting an analytical approach which can lead to serious errors in stress analysis in circumstances arising in armaments design problems, perhaps the most significant development described in this report is that associated with materials exhibiting strain-induced anisotropy. The phenomenon considered includes the Bauschinger effect exhibited by many metals subjected to large strains. For large deformations, what was recently

considered the correct formulation for stress analysis introduces huge errors in stress evaluation due to incorrect representation of the influence of rotation on the development of plastic anisotropy. A promising first approach towards correcting the error has been devised and tested in simple shear deformation. Because of its importance in applications it is imperative that investigations of this problem, which first surfaced at a workshop co-sponsored by the Army Research Office, be continued, particularly in view of the controversy concerning the correct approach to its solution described in the section: STRESS ANALYSIS FOR PLASTIC-STRAIN-INDUCED ANISOTROPY.

SCIENTIFIC PERSONNEL ENGAGED ON THE PROJECT

A. Agah-Tehrani, Graduate Student

K. Chung, Graduate Student (will complete his Ph.D. at Stanford University during a visit there in January 1984)

E. H. Lee, Professor

R. L. Mallett, Professor

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APPENDIX DECOMPOSITION OF SIMPLE SHEAR INTO PURE DEFORMATION AND ROTATION (POLAR DECOMPOSITION)

Consider simple shear strain γ which displaces the point A with coordinates $(a,1)$, to B with coordinates $(a+\gamma,1)$ as shown in the figure. The

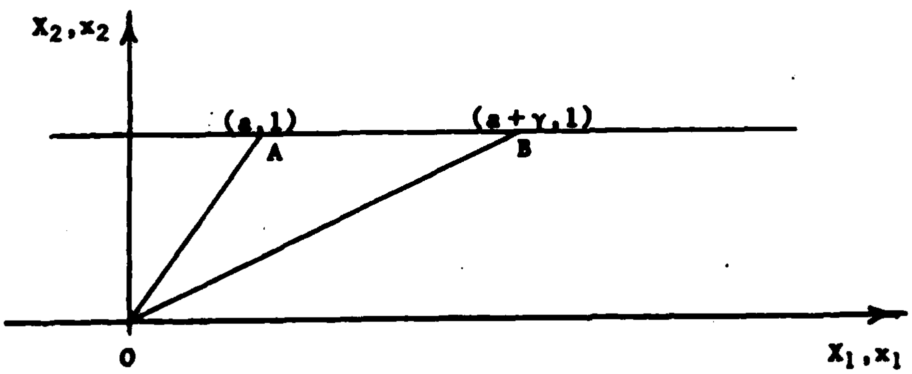

Fig. 3. - Stretch of a material line.

stretch ratio λ of the material line OA is:

$$\lambda = \sqrt{1+(a+\gamma)^2} \,/\, \sqrt{1+a^2} \tag{1}$$

By differentiating λ with respect to a at fixed γ the stretch ratio is found to attain maximum and minimum values for

$$a = -\gamma/2 \pm \sqrt{1+(\gamma/2)^2} \tag{2}$$

The initial lines (OA) for the two values of a are found to be orthogonal as also are the deformed material lines OB. These values thus determine the principal stretches which correspond to a square being deformed into a rectangle as shown in Fig. 2 for $\gamma = 4$. Figure 4 shows the positions of the corresponding material lines OA_i and OB_i, $i = 1,2$. The initial positions are

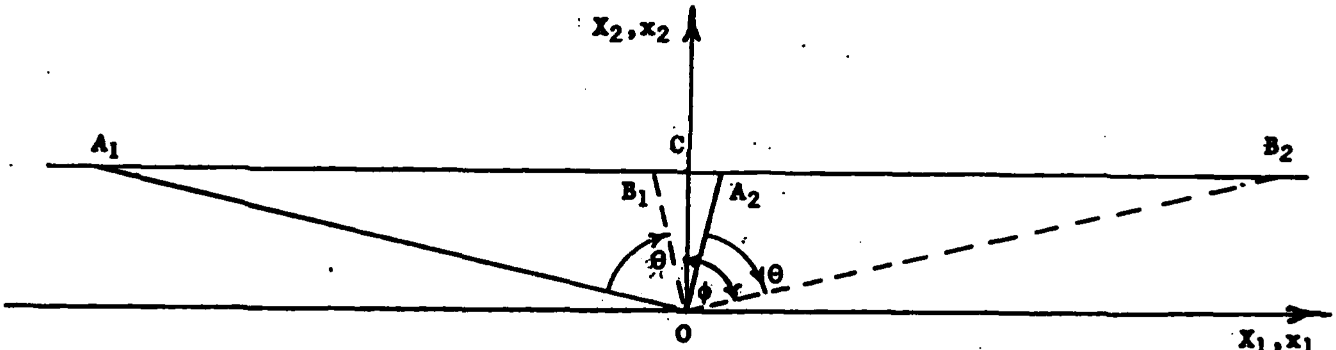

Fig. 4. - Deformation of principal directions

drawn in full lines, the final in broken lines. The polar decomposition rotation θ is given by

$$\tan\theta = \tan(B_2\hat{O}C - A_2\hat{O}C) = \gamma/2 \tag{3}$$

The final position of the elongated principal direction is given by ϕ:

$$\tan\phi = \gamma/2 + \sqrt{1+(\gamma/2)^2} \tag{4}$$

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